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ON ESTIMATION OF THE HEDGE RATIO IN MANAGEMENT OF PRICE RISK OF AGRICULTURAL COMMODITIES

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Abstract

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It is often the case that producers and users of agricultural commodities hedge the exposition in a commodity (e.g. an expected purchase, expected sale, commodity held on stock, commodity being processed) with a derivative position in another, highly correlated market variable. The main reason is that while the commodity traded in a public derivative market, such as CBOT, is highly standardized, spot markets exist for a wide variety of types of that commodity (e.g. different cultivated species, different classes, different qualities etc.). In general, prices of various types of the commodity traded in the spot markets can be regarded as functions of the price of the standardized commodity. When an open position in the non-standardized type y of the commodity is hedged, using the standardized derivative x, the basic risk-management issue is to estimate the hedge ratio between y and x (or, more generally, the long-term average price relationship between y and x) properly in order to minimize risk and subsequent losses. The hedge ratio is usually estimated from historical market data. We address the question of long-term stability of the hedge ratio and propose a new method for estimation of the hedge ratio allowing an improvement of the hedging relationship using stability analysis. We illustrate the method by an example. We also present a user-friendly visualization technique for the method.

Key words: Market-traded commodities; agricultural futures markets; hedge ratio; standardized commodities; stability analysis

Introduction

Companies producing, consuming, processing or storing agricultural commodities often face the risk of changes in market prices. They implement various hedging strategies to minimize that risk. For example, a wheat producer may wish to lock future prices of wheat to protect her/him from a decline in spot prices. On the other hand, a consumer may wish to protect against an increase in spot prices. A wheat processing company, whose inputs and outputs are sensitive to wheat prices, might wish to hedge the value of wheat held on stock. The most usual way of getting protection is to take a derivative (forward) position in the commodity offsetting the open spot position.

This basic strategy does not apply to the agricultural commodities only; the same strategy is used e.g. in the case of foreign exchange risk. There is a significant difference: while the FX market is homogenous, an agricultural commodity usually exists in various types, cultivated species or qualities. However, the markets where futures (or other derivatives) suitable for hedging of the commodity risk are traded admit contracts with highly standardized commodities only. Now the main

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problem arises. Say that a producer of Class A wheat wishes to hedge her/his open position (i.e., future sales of Class A wheat) via traded futures. Only standardized contracts, say Class B wheat futures are available in the market. It is clear that prices of the two variables are highly correlated. Hence, it makes sense that the producer hedges her/his position in Class A wheat using Class B wheat futures. The question is what volume of Class B wheat futures shall be bought/ sold to hedge the open position in one ton of Class A wheat? That ratio is known as *the hedge ratio*. The main aim of this text is to present a new method for its estimation.

As an example of standardization, we can look at Chicago Board of Trade (CBOT) Rulebook, Chapter 14 Wheat Futures, Rule 14104 Grades/Grade Differentials. *At contract price, the following types of wheat are delivered: №2 Soft Red Winter, №2 Hard Red Winter, №2 Dark Northern Spring, №2 Northern Spring. At ¢3 premium, the following types are delivered: №1 Soft Red Winter, №1 Hard Red Winter, №1 Dark Northern Spring, №1 Northern Spring. Wheat which contains moisture in excess of 13.5% is not deliverable. A taker of delivery of wheat shall have the option to request in writing load-out of wheat which contains no more than four parts per million of deoxynivalenol (vomitoxin). All wheat shipping certificates shall be marked as either 2 parts per million (ppm) deoxynivalenol (vomitoxin), 3ppm vomitoxin, or 4ppm vomitoxin. Shipping certificates marked as 2ppm vomitoxin shall be delivered at contract price, while shipping certificates marked as 3ppm shall be delivered at a ¢12 per bushel discount and shipping certificates marked as 4ppm shall be delivered at a ¢24 per bushel discount.* Moreover, further conditions are often specified, e.g. delivery terms such as FOB Gulf of Mexico.

The cited Rule in fact establishes a fixed relationship between prices of different wheat types. For example, if x denotes the price of №2 Soft Red Winter with 2ppm vomitoxin and y denotes the price of №1 Soft Red Winter with 4ppm vomitoxin, then we have a linear relationship between y and x in the form

$$y = x - ¢21. \qquad (1)$$

This is just a trivial example of a linear relationship between the prices of two (slightly) different, but perfectly correlated commodities y and x; the perfect correlation is a consequence of the cited Rule.

However, in practice, more types of wheat are traded in OTC markets for which a fixed relationship of the type (1) is explicitly stated by no Rule and holds only approximately. For example, the market maker has excluded wheatcontaining moisture exceeding 13.5%. Prices of such a commodity are known from spot OTC markets, but a position in that commodity cannot be directly hedged via a CBOT futures contract.

Hedging then can be done indirectly: a long-term relationship of the type (1) between the price of a commodity y, for which futures are not available, and the futures on commodity x can be estimated from historical spot prices of y and quotes of x. Usually, the trivial form (1) is too restrictive: a more general form

$$y = \alpha + \lambda x + \text{random error},$$

where α and λ are parameters, will be used in the next section.

The parameter λ, called hedge ratio, is essential when the price of the commodity y is, in the long run, a *multiple* of the price of x. To give an example from a non-agricultural market: in the airline industry, it is well known that kerosene prices are approximately equal to 130% of Brent crude oil. This example shows that the method of this text can be useful also in other industries.

The main problem. When such a hedging relationship is established, the crucial question is to estimate the hedge ratio properly. For example, shall one ton of y be hedged using futures with the volume of 1.2, 1.3 or 1.4 tons of x? The hedge ratio, i.e. the ratio between a unit of y and the number of units of x, is the crucial factor determining the quality of the hedging relationship. If the hedge ratio is selected too low, then an unhedged position in y remains open, which may result in significant losses. On the other hand, if the hedge ratio is selected too high, then a new speculative position in x originates, which may result in significant losses as well. The risk-management aim is to select the hedge ratio in order the position be fully hedged. In the next sections, we shall present a method for its estimation.

Model

The situation is: an open position in a variable y, which is not traded in futures market, is to be hedged using a derivative position in another variable x, which is available at the futures market. The task is to determine 'the best possible' hedge ratio between x and y at the commencement of the hedging relationship.

The hedge ratio λ is estimated from historical data. A long-term relationship between y and x is usually assumed to be of a form as

$\Delta y_t = \lambda \Delta x_t + \varepsilon_t$,
$\Delta y_t = \alpha + \lambda \Delta x_t + \varepsilon_t$,
$y_t = \lambda x_t + \varepsilon_t$,
$y_t = \alpha + \lambda x_t + \varepsilon_t$,
$\Delta \log y_t = \lambda \Delta \log x_t + \varepsilon_t$,
$\Delta \log y_t = \alpha + \lambda \Delta \log x_t + \varepsilon_t$,

where t is the index of time, Δ denotes the difference operator and ε_t is the random error.

Let ν_t be homoskedastic with unit variance. The random errors ε_t are usually assumed in one of the forms

$\varepsilon_t = \sigma \nu_t$,
$\varepsilon_t = x_t \sigma \nu_t$,
$\varepsilon_t = y_t \sigma \nu_t$,

where $\sigma > 0$ is a parameter. As an example, important in practice, we shall assume the model

$$y_t = \alpha + \lambda x_t + x_t \sigma \nu_t; \qquad (2)$$

however, the method presented further is applicable for other models as well. (The fact that variance of the error term is proportional to the price level x_t is a traditional feature of financial time series.)

The hedge ratio λ can be estimated as the absolute term in the homoskedastic model

$$\frac{y_t}{x_t} = \lambda + \alpha \frac{1}{x_t} + \sigma \nu_t \qquad (3)$$

which is equivalent to (2).

We shall assume that ν_t's are such that the model (3) can be estimated with Ordinary Least Squares (OLS).

Estimation of the Hedge Ratio

Prices of commodities, both on OTC and futures markets, are quoted for several decades. We assume that daily prices x_t and y_t with very long history are available. Our aim is to estimate λ in (3) as precisely as possible using the historical data. It is well known that the variance of $\hat{\lambda}$ (where $\hat{\lambda}$ is the OLS-estimator of λ) decreases with the number of observations. Hence, if we assume a long-term relationship (3), to get the most exact estimate of λ we should use all the historical data available.

However, on the other hand, it is doubtful whether y:x ratios from say 1950's are relevant for estimation of the *contemporary* hedge ratio which is essential for *contemporary* hedging of y with x-futures. It is not clear whether the assumption that the relationship (3) remains stable over decades is reasonable. It seems more realistic to assume that the relationship (3) is stable in the short run (i.e., say months of a few years, which is the time horizon for which the hedging relationship is usually designed) while in the end it may be subject to changes. The short-run stability assumption is crucial; otherwise, the hedging would not make sense.

The main question now arises: *how long history of data shall be taken into account* when estimating the hedge ratio λ in (3) in order

- to minimize the variance of the estimator (i.e., to estimate λ as exactly as possible) and, simultaneously,
- to avoid estimation bias arising from possible instability of the value of λ in the long run.

We shall propose a method to deal with this problem. The problem of determination of the hedge ratio has also been addressed, from a different perspective, in Černý and Hladík (2010), Choudry (2009), Hladík and Černý (2010, 2012), McMillan (2005), Lien and Shrestka (2008). We shall introduce a method, which is partially motivated by the approach applied in Černý (2008).

A Statistic for Testing Stability

Assume that the set of historical data $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ is available. Let us test the hypothesis

H: the relationship (3) is valid for all $t \in \{1, \ldots, n\}$

against the alternative

A: there is a time $\tau \in \{3, \ldots, n-3\}$ such that

$$\frac{y_t}{x_t} = \begin{cases} \lambda_0 + \alpha_0 \cdot \dfrac{1}{x_t} + \sigma \nu_t & \text{for } t \in \{1, \ldots, \tau\}, \\ \lambda_1 + \alpha_1 \cdot \dfrac{1}{x_t} + \sigma \nu_t & \text{for } t \in \{\tau + 1, \ldots, n\}, \end{cases}$$

where $(\alpha_0, \lambda_0) \neq (\alpha_1, \lambda_1)$ and τ are unknown parameters. Assuming that ν_t are $N(0, 1)$ independent, we can construct the log-likelihood ratio

$$\begin{aligned} L &= \ln \frac{f_A}{f_H} \\ &= \ln \prod_{t=1}^{\tau} \frac{1}{\sigma \cdot \sqrt{2\pi}} \exp\left(-\frac{(y_t - \lambda_0 x_t - \alpha_0)^2}{2x_t^2 \sigma^2} \right) \\ &+ \ln \prod_{t=\tau+1}^{n} \frac{1}{\sigma \cdot \sqrt{2\pi}} \exp\left(-\frac{(y_t - \lambda_1 x_t - \alpha_1)^2}{2x_t^2 \sigma^2} \right) \\ &- \ln \prod_{t=1}^{n} \frac{1}{\sigma \cdot \sqrt{2\pi}} \exp\left(-\frac{(y_t - \lambda x_t - \alpha)^2}{2x_t^2 \sigma^2} \right) \\ &= \frac{1}{2\sigma^2} \left[\sum_{t=1}^{n} \frac{(y_t - \lambda x_t - \alpha)^2}{x_t^2} - \sum_{t=1}^{\tau} \frac{(y_t - \lambda_0 x_t - \alpha_0)^2}{x_t^2} - \sum_{t=\tau+1}^{n} \frac{(y_t - \lambda_1 x_t - \alpha_1)^2}{x_t^2} \right] \end{aligned}$$

where f_A and f_H denote the joint distribution of $\frac{y_t}{x_t}$ under A and H, respectively. If we assume that τ is fixed, we get the log-likelihood test for the existence of change in the regression relationship in time τ of the form

$$U_\tau = \frac{RSS_{1:n} - RSS_{1:\tau} - RSS_{\tau+1:n}}{2\sigma^2},$$

if the standard error σ is known, or

$$U_\tau = \frac{n-2}{2} \cdot \frac{RSS_{1:n} - RSS_{1:\tau} - RSS_{\tau+1:n}}{RSS_{1:n}},$$

if σ is unknown (which is the most frequent case in practice). The symbol $RSS_{i:j}$ stands for the residual sum of squares from OLS-estimated regression

$$\frac{y_t}{x_t} = \lambda + \alpha \frac{1}{x_t} + \sigma \nu_t$$

using the data set $t \in \{i, i + 1, \ldots, j\}$. More precisely, denoting

$$\mathbf{x}_t = (1 \quad \tfrac{1}{x_t}), \quad \mathbf{X}_{i:j} = \begin{pmatrix} \mathbf{x}_i \\ \mathbf{x}_{i+1} \\ \vdots \\ \mathbf{x}_j \end{pmatrix}, \quad \mathbf{z}_{i:j} = \begin{pmatrix} \frac{y_i}{x_i} \\ \frac{y_{i+1}}{x_{i+1}} \\ \vdots \\ \frac{y_j}{x_j} \end{pmatrix},$$

it holds

$$RSS_{i:j} = \| (\mathbf{I} - \mathbf{X}_{i:j} (\mathbf{X}_{i:j}^{\mathrm{T}} \mathbf{X}_{i:j})^{-1} \mathbf{X}_{i:j}^{\mathrm{T}}) \mathbf{z}_{i:j} \|^2,$$

where $\mathbf{I}$ denotes the unit matrix and $\| \cdot \|$ denotes the L_2-norm.

Changing normalization, instead of U_τ we will use an equivalent statistic

$$V_\tau = \frac{RSS_{1:n} - RSS_{1:\tau} - RSS_{\tau+1:n}}{RSS_{1:n}}.$$

The reason for preferring V_τ to U_τ is purely technical and will be apparent later. Relaxing the assumption that τ is fixed we obtain the statistic

$$V = \max_{t \in \{3, \ldots, n-3\}} V_t. \tag{4}$$

We will also need the statistic V applied to a subset $\{i, i + 1, \ldots, j\}$ of the set of all observations $\{1, \ldots, n\}$. Thus it will be useful to denote

$$V_{i:j} = \max_{t \in \{i+2, \ldots, j-3\}} V_t^{i:j}, \tag{5}$$

where

$$V_t^{i:j} = \frac{RSS_{i:j} - RSS_{i:t} - RSS_{t+1:j}}{RSS_{i:j}}.$$

We shall need critical values for the statistic V (or $V_{i:j}$) under H. The statistic V, being the maximum of dependent $B_{1,\, n/2-2}$-distributed random variables, has a complicated distribution; in fact, an exact formula is not known.

Fortunately, the statistic V is essentially the same statistic as investigated by Worsley (1983); this is why we have used V_t instead of U_t in (4). Worsley derived a Bonferroni-type approximation (see also Černý (2011)) of the distribution of V in the form

$$W(z) = \Pr[V \leq z] \approx B_{1,\, n/2-2}(z) - \frac{2\beta_{3/2,\, n/2-1}}{\pi(n-2)} \times \cdot \left(\sum_{t=2}^{n-3} \xi_t - \frac{1}{6} \left(\frac{n-5}{6} \cdot \frac{z}{1-z} - 1 \right) \cdot \sum_{t=2}^{n-3} \xi_t^3 \right),$$

where

$$\xi_t = \sqrt{\mathbf{x}_{t+1}^{\mathrm{T}} (\mathbf{X}_{t+1:n}^{\mathrm{T}} \mathbf{X}_{t+1:n})^{-1} \mathbf{X}_{1:n}^{\mathrm{T}} \mathbf{X}_{1:n} (\mathbf{X}_{1:t+1}^{\mathrm{T}} \mathbf{X}_{1:t+1})^{-1} \mathbf{x}_{t+1}^{\mathrm{T}}}.$$

Here, B and β denote the cdf and the pdf of the corresponding beta-distribution, resp. Using binary search over $W(z)$, it is computationally feasible to derive the z_0-quantile for V given the level z_0. This z_0-quantile will be referred to as the *Worsley's z_0-critical value*.

If we need to work with the test (5) instead of (4), i.e. if we are restricted to a subset of observations, the Worsley's approximation gets the form

$$W_{i:j}(z) = \Pr[V_{i:j} \leq z] \approx B_{1,(j-i+1)/2-2}(z) - \frac{2\beta_{3/2,(j-i+1)/2-1}}{\pi(j-i-1)} \cdot \left(\sum_{t=i+1}^{j-3} \xi_t^{i:j} - \frac{1}{6}\left(\frac{j-i-4}{6} \cdot \frac{z}{1-z} - 1 \right) \cdot \sum_{t=i+1}^{j-3} (\xi_t^{i:j})^3 \right),$$

where

$$\xi_t^{i:j} = \sqrt{\mathbf{x}_{t+1}^{\mathrm{T}} (\mathbf{X}_{t+1:j}^{\mathrm{T}} \mathbf{X}_{t+1:j})^{-1} \mathbf{X}_{i:j}^{\mathrm{T}} \mathbf{X}_{i:j} (\mathbf{X}_{i:t+1}^{\mathrm{T}} \mathbf{X}_{i:t+1})^{-1} \mathbf{x}_{t+1}^{\mathrm{T}}}.$$

The z_0-quantile derived by binary search over $W_{i:j}(z)$ will be denoted $W_{i:j}^{-1}(z)$.

If H is rejected, then (4) also suggests a natural estimator of the unknown value τ of the form

$$\tau = \arg\max_{t \in \{3,\dots,n-3\}} V_t,$$

or, in the restricted form,

$$\grave{\tau}_{i:j} = \arg\max_{t \in \{i+2,\dots,j-3\}} V_t^{i:j}.$$

Our Method

Now we are ready to present the method for estimation of the hedge ratio when hedging the open position in the commodity y, which is traded only in the spot markets, with the commodity x for which futures are available. Assume that the historical data $x_1, \dots, x_n$ and $y_1, \dots, y_n$ are sorted in the way that (x_1, y_1) are the most recent (say, today's) prices and (x_n, y_n) are the endmost prices (say, quoted several decades ago).

The method estimates the hedge ratio from a set of historical data in a way that

- it maximizes the length of the historical period in which the hedge ratio λ appears to be stable (this is essential for achieving low variance of $\hat{\lambda}$, i.e. for achieving an estimate which is as precise as possible); and
- it excludes the history in which the value of λ appears to be different from its contemporary value (this is essential for avoiding estimation bias).

The method can be summarized as the following algorithm. Assume that the test level z_0 is fixed (say at 5% or 1%).

{1} **for** $t := 20$ **to** n **do**
{2} **if** $V_{1:t} > W_{1:t}^{-1}(z_0)$ **then** stop and output $\tau_{\text{out}} := \grave{\tau}_{1:t}$
{3} **end if**
{4} **next** t
{5} stop and report "the hedge ratio is stable over the whole data set".

If the algorithm terminates in {5} with a value τ_{out}, we get the information that we shall use the dataset {1, …, τ_{out}} for estimation of the hedge ratio; the data {$\tau_{\text{out}} + 1$, …, n} are omitted. If the algorithm terminates in {4}, then we can use entire data set.

Remark. The chosen value 20 in {1} is arbitrary; any other reasonable value could be used.

Remark. It is suitable to take into account the fact that the estimator $\tau_{1:t}$ in {2} could have estimated the true point of the most recent change in the hedge ratio inexactly. To the author's knowledge, the exact distribution of τ under A is not known. It seems to be reasonable to get over the loss of a reasonable number of observations, say m (determined heuristically), and estimate the hedge ratio using the data set {1, …, $\tau_{\text{out}} - m$} instead of {1, …, τ_{out}}. Giving up m observations, we increase the variance of the estimator of the hedge ratio; but this is offset by the fact that we avoid the possible bias resulting from an inexact estimate of the point of change. (A recommendation, how m should be chosen in practice, is subject to further research.)

The crucial property of the method {1} – {5} is that it is *insensitive to the length of history of data available*. (This is a well-known problem in econometrics: if two analysts have data sets for slightly different periods, they can achieve very different results). The method processes the data (x_t, y_t) from the most recent observations towards older observations. As soon as the data are sufficient to detect a point of change in the hedge ratio, the process is stopped and the history preceding the most recent point of change is dropped. For example, assume that we perform the analysis in Jan 2011. Assume that in May 2010 there was such a point of change. Then the method (if it detects the point of change correctly) only needs the data for the period, say, Jan 2010 – Jan 2011. That is, the method stops in {2} with t = Jan 2010. The data from the period Jan – May 2010 brings the information that the hedge ratio had been different before May 2010. But the data preceding Jan 2010 are not processed at all; hence, the results of the method are the same regardless of whether the entire data set starts in 2000, in 1990 or for example in 1930. (Many statistical methods process the entire available data set globally. Their results can thus be affected by very old observations. We have avoided this undesirable property.)

Visualization of the Method, Stability of τ_{out} and an Example

The method can be visualized in the following way. We plot the processes

$$\tilde{V}_t = t \cdot V_{1:t}, \quad \tilde{W}_t = t \cdot W_{1:t}^{-1}(z_0), \quad \breve{\tau}_t = \hat{\tau}_{1:t}$$

for $t = 20, 21, \ldots$ with z_0 = 5%-level and z_0 = 1%-level. (The scaling factor t in the definition of $\tilde{V}_t$ and $\tilde{W}_t$ has been added, without loss of generality, to make Figure 2 more transparent.) Such a plot also shows how stable the value τ_{out} output in {2} is. (We say that the value τ_{out} is *fully stable* if, for any t and t' for which the conditions $V_{1:t} > W_{1:t}^{-1}(z_0)$ and $V_{1:t'} > W_{1:t'}^{-1}(z_0)$ are met, it holds $\breve{\tau}_{1:t} = \breve{\tau}_{1:t'}$. Of course, by the stochastic nature of data, we cannot expect full stability; but we roughly say that the value τ_{out} is stable if it would not change too much if we did not stop in {2} and iterated further. In other words: when the **if**-condition in {2} is met, we do not stop the algorithm and iterate further (say, up to $t = n$). Whenever the **if**-condition is met, we plot $\hat{\tau}_{1:t}$ in a graph (with t on the x-axis). If the resulting graph resembles a constant function, then the estimate is stable.)

In Figure 2, results of a simulated example are shown. We generated a trajectory of x_t for $t = 1, \ldots, 500$ (which corresponds to 2 years if 1 year = 250 business days) as a lognormal random walk varying between \$20 and \$85, see Figure 1. The process y_t was simulated using (3) with $\nu_t \sim N(0, 1)$ independent and

$$\sigma = 0.1, \quad \alpha = 0, \quad \lambda = \begin{cases} 1.3 & \text{for } t \in \{1, \ldots, 169\}, \\ 1.4 & \text{for } t \in \{170, \ldots, 500\}. \end{cases} \qquad (6)$$

Observe that the variance is quite high: if the price of x is \$100, the standard error is \$10.

In Figure 2 it is apparent that the procedure {1} – {5} detects $\tau_{out} = 134$, which is an inexact estimate (by (6), the point of change appeared 169 days ago).

If we don't stop in the step {2} when the 1% level is first exceeded and iterate further, we arrive at the estimate $\tau_{out} = 149$ (which is a value closer to the true value 169).

It can be seen in Figure 2 that the estimate $\tau_{out} = 149$ is stable. Hence, the method suggests to estimate the hedge ratio either using last 133 or last 148 observations, the remaining ("old") ones being omitted.

We know that the true value of the contemporary hedge ratio is 1.4. The resulting OLS-estimates are

$$\hat{\lambda}_{\text{from data } t \in \{1, \ldots, 133\}} = 1.43 \text{ and } \hat{\lambda}_{\text{from data } t \in \{1, \ldots, 148\}} = 1.46.$$

If we estimate the hedge ratio from the entire data set, we obtain a much worse value

$$\hat{\lambda}_{\text{from data } t \in \{1, \ldots, 500\}} = 1.23,$$

which is clearly biased by the "old" history when the value of the hedge ratio was 1.3. If the value 1.23 had been used, only 88% (= 1.23 ÷ 1.4) of the open position in y would be hedged (on average), while with the presented method we get an "over-hedged" position of 102% (= 1.43 ÷ 1.4) or 104% (= 1.46 ÷ 1.4), respectively.

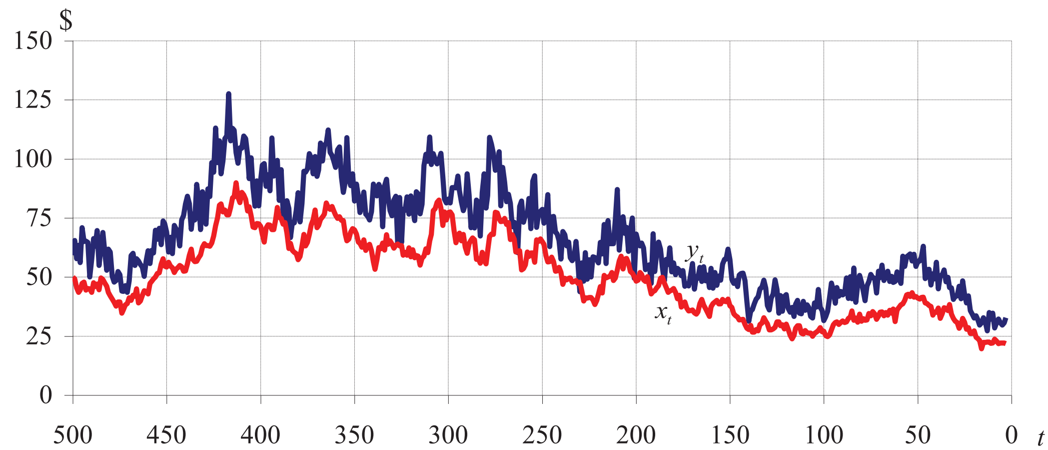

Fig. 1. Simulated evolution of prices of the hedged commodity (y_t) and the hedging commodity (x_t) as a function of time (t)

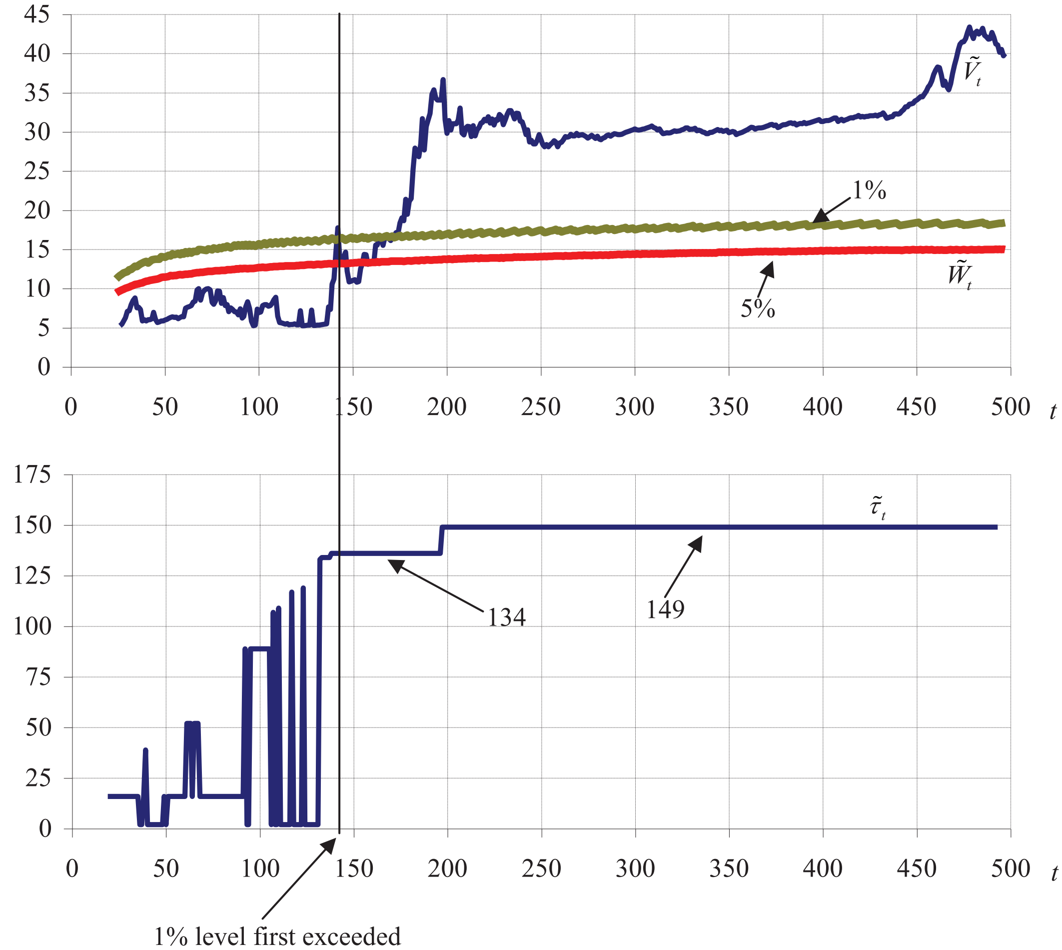

Fig. 2. The process $\tilde{V}_t$, Worsley's 5% and 1% critical values ($\tilde{W}_t$) and the process $\tilde{\tau}_t$

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