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FRACTAL ANALYSIS OF RHIZOME GROWTH PATTERNS OF *SCIRPUS GROSSUS* L. ON PEAT AND PADDY SOILS

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Abstract

MAJRASHI, A., B. BIN BAKAR, M. MONERUZZAMAN KHANDAKER, A. NASRULHAQ BOYCE and S. V. MUNIANDY, 2013. Fractal analysis of rhizome growth patterns of *Scirpus grossus* L. on peat and paddy soils. *Bulg. J. Agric. Sci.,* 19: 1319-1326

This study describes the use of fractal analysis to describe root system in rhizomatous plant, *Scirpus grossus* L. Root network spread and filling factor of rhizomatous roots are compared with fertilizer applications under different soil types. Fractal analysis allows the structural complexity of such associations to be compared between plant communities, with regard to their potentials for soil resource acquisition and utilization. The NPK fertilizer application at 100:30:30 ha^{-1} resulted in more robust aerial plant growth with *ca.* 253.5 ramets m^{-2} (mean dry aerial bio mass of 23.2 g plant^{-1}) compared with 235.6 ramets m^{-2} (16.3 g plant^{-1}) in unfertilized peat soils 24 weeks after planting of the mother plant. The parallel figures for plants growing on paddy soils of the Jawa series were *ca.* 97.08 ramets m^{-1} (12.19 g plant^{-1}) (fertilized paddy soils) and 83.67 ramets m^{-1} (10.89 g plant^{-1}) (unfertilized paddy soils) 24 weeks after planting of the mother plant. Mean ramets mortality was significantly higher in unfertilized paddy soils at 121.3 ramets m^{-2}, while in the fertilized paddy soils this was only 34.7 ramets m^{-2}, resulting respective net populations of *ca.* 218.8 ramets m^{-2} and 114.3 ramets m^{-2} in fertilized and unfertilized plots. In paddy soils mean ramets mortality in unfertilized paddy soils was *ca.* 8.58 ramets m^{-2}, while this was only *ca.* 5.67 ramets m$^{-2,}$ leading to the respective resultant net populations of 91.41m^{-2} and 75.09 ramets m^{-2}. Flowering set in earlier among ramets in fertilized peat soils with 103.2 ramets m^{-2}*vis-a-vis* 77.5 ramets m^{-2}, 24 weeks after transplanting of the mother plant in unfertilized soil. This method allows the structural complexity of such associations to be compared between plant communities, with regard to their potentials for soil resource acquisition and utilization. In peat soil, distinct and partly not significant differences are found (fractal dimension between 1.52 ± 0.53 and 1.50 ± 0.59) in unfertilized and fertilized plots and in paddy soil, fractal dimension between 1.53 ± 0.55 and 1.52 ± 0.49) in unfertilized and fertilized plots. We found distinct and partly not significant differences be compared between plant in peat and paddy soils when analysing many small units of a complex root system association. In larger plant communities, a broad variety of below-ground structures is recorded in its entirety, integrating the specific features of single sub-structures. In that way, extreme fractal dimensions are lost and the diversity decreases. Therefore, the analysis of larger units of root system associations provides a general knowledge of the complexity of root system structures for heterogeneous plant communities.

Key words: fractal dimension, fertilizer application, modular growth, *Scirpus grossus*

Introduction

Scirpus grossus L. is a principal rhizomatous weed in the rice fields, drainage and irrigation canals, riverbanks, abandoned rice fields and wasteland in Malaysia. Rhizomatous plants grow and reproduce clonally by rhizomes. Clonal branches are formed from the reiteration of the basic units, while flowers and inflorescences come from the reiteration of units bearing modified leaves. The population dynamics of many rhizomatous plants is dominated more by the flux of clonal modules. The ability of a single genotype to form fragmented phenotypes is just one of the variants in the life patterns of modular organism (Harper and Bell, 1979; Baki, 1986). The process of new growth is often subjected to different pressures, including the change in soil nutrients, and

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resource capture ability among individual plants and their modules. *Scirpus grossus* is a pan-tropical weed in the rice fields, drainage and irrigation canals, riverbanks, abandoned rice fields and wastelands in Malaysia and elsewhere. There is a paucity of information on the population biology of this scourge in the literature. We report on the allometry, modular dynamics, and spatio-temporal growth patterns of aerial plant modules, and sub-terranean rhizome populations of this scourge in fertilized and unfertilized peat and paddy soils.

Root growth is effected by the acquisition, distribution, and consumption of water and nutrients by plants. We want to understand the root branching complexity. Mathematical models using fractals and computers can be applied to accurately understand the growth and form complexity of plant root systems. Because these we can useing the Fractal Dimensions (FD) software's box-counting method of fractal analyses. The data from the fractal counting were tabulated and plotted on a log-log plot graph (Klarizze, 2005). Fractals are unusual geometric structures that can be used to analyze many biologic structures not amenable to conventional analysis (Richardson and Gillepsy, 2000). It was suggested that when roots develop under favorable conditions, FD is a good indicator for estimating the system's size and root branching (Klarizze, 2005).

A number of researchers have demonstrated that fractal analysis may be biologically relevant. Many aspects of morphological and physiological variation have been associated with variations in FD. As roots grow, the FD increases (Fitter and Stickland, 1992; Lynch and van Beem, 1993; Nielsen et al., 1998). Fractal dimension has been correlated with root topology (Fitter and Stickland, 1992) and root architecture (Nielsen et al., 1997). Differences in FD have been noted among four species of dicots and monocots (Fitter and Stickland, 1992), as well as among genotypes of sorghum, rice and common bean (Izumi et al., 1995; Masi and Maranville, 1998; Nielsen et al., 1998). Genotypic variation of root mass and root: shoot ratio was correlated with variation in FD of roots (Masi and Maranville, 1998).

Nielsen et al. (1997) have successfully applied fractal analysis to a computer model of bean roots. They have also demonstrated that the true three dimensional fractal dimension ($D3$) is tightly correlated with planar ($D2$) and linear ($D1$) fractal dimensions. This attribute may prove to be a cardinal factor in the application of fractal analysis to soil-grown root systems because $D2$ can be determined from data collected by trenching. Many studies used this technique for several targets. Including the architecture of the root system is also well known to be a major determinant of root functions in the acquisition of soil resources such as nutrients and water (Lynch, 1995; Yamauchi et al., 1996; Fitter, 2002). Fractal geometry is being widely applied to assess the root system architecture and the distribution of root systems in soils (Fitter and Stickland, 1992; Berntson et al., 1997; Lynch and van Beem, 1993; Tatsumi et al., 1989; Tatsumi, 1995, 2001; Masi and Maranville, 1998; Walk et al., 2004; Dannowski and Block, 2005). Fractal geometry is a system of geometry that is more suitable for the description of complex natural objects than standard Euclidian geometry (Mandelbrot, 1983). A fractal is an object having a non-integer dimension. Root systems also have self-similarity and are considered as the approximate fractal objects over a finite range of scales (Tatsumi et al., 1989). Fractal analysis in root biology often typically utilizes box-counting method and the equation: $N(r) = Kr^{-D}$ is obtained (Tatsumi et al., 1989; Tatsumi, 2001; Walk et al., 2004), where r is the length of the box side, and $N(r)$ is the number of boxes of size r needed to cover the object. In terms of fractal analysis, the equation: $N(r) = Kr^{-D}$ is transformed to the regression of log of $N(r)$ intersected by roots vs. r levels. The slope (D) and intercept log K are computed. D is the fractal dimension (FD), and log K is associated with fractal abundance (FA). The FD is closely related with the branching pattern of roots, while the FA with the volume of space explored by roots (Tatsumi et al., 1989; Tatsumi, 2001; Walk et al., 2004).

Diffusion-limited elements, such as P, move slowly through soil (Schenk and Barber, 1979), so soil volumes in the immediate vicinity of roots are often depleted of P, while bulk soil 1±2 cm away has P concentrations that remain largely unchanged. The soil that has had P concentration decreased by root uptake is referred to as the depletion volume, the size of which is proportional to the rate of diffusion of P through the soil. Some soil is depleted of P by more than one root from a single plant. The volume of soil explored by multiple roots is defined as competition volume. Since competition volume increases as the root system grows, it is often useful to calculate relative competition, which in this report is competition volume divided by depletion volume. Using these concepts of depletion and competition, P efficiency can be defined as minimization of relative competition. This definition has been used in geometric modelling to evaluate the P efficiency of contrasting root architectures (Ge et al., 2000; Rubio et al., 2001).

Materials and Methods

Plant establishment and care

Synthetic populations of *S. grossus* were established on peat and paddy soils in the Malaysian Agriculture Research Development Institute (MARDI) Research Station of Jalan Kebun, Klang, and Tanjung Karang, Malaysia, respectively for 24 weeks commencing on 24 February 2010. Both stations had more than 50-year history of NPK fertilizer applications ranging from 80:30:20 to 100:30:30 depending on the crops planted. Cohorts of young ramets at 2-3-leaf of *S. grossus* were obtained from rice fields of Tanjung Karang, Selangor. Each ramet was planted in the centre of a plot measuring 2m x 2m, previously demarcated and lined with 5cm x 5cm grids and sub-plots (Figure 1). Fertilizer applications with Nitrophoska Blue Special NPK fertilizer at the rate of 100:30:30 were made one week prior to planting. A set of 3 replicated plotswith fertilizer application was allocated with while another 3 sets served as a control. A standard plant care was instituted against pests and weeds.

Data acquisition and management

Clonal and reproductive growth parameters were recorded weekly in each plot, viz: (i) Number of emerged plants, their locations, and heights (ii) Number of dead plants and their positions.

(iii) Mapping of actual positions of emerged/dead ramets and in the 2m x2m quadrates. (iv) At harvested after 24 weeks mapping of subterranean rhizomes in the 2m x2m quadrates. The plants were harvested after 24 weeks by dismembering them into leaf, stem, and inflorescence components, and their dry weights were determined. The sub-terranean rhizomes remained intact by ensuring that no rhizome damage was inflicted during harvest of the aerial plant parts.

Measuringinter-nodal lengths of each rhizome mapped the exposed rhizomes, by noting the precise positions of the harvest plants. These data were transferred into the data logger, and together with the weekly data on the precise spatio-temporal positions of emerged plants of *S. grossus*, computer generated subterranean rhizome architectures were produced. The computer program used was AutoCAD 10 (model from Autodesk C. From USA).The appropriate data were subjected to ANOVA and their treatment means were tested for significant difference, if any, using HSD and *t*-tests (Zar, 2006).

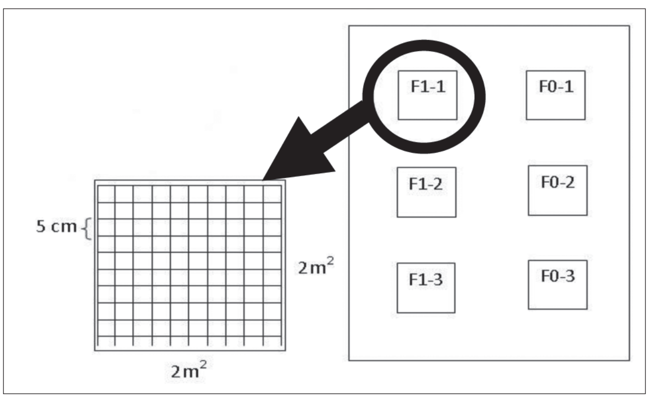

Fig. 1. Experimental design and quadrats arrangement of *Scirpus grossus* in MARDI Research Stations, Jalan Kebun and Tanjung Karang, Selangor, Malaysia. F_0-No fertilizer application; F_1, NPK applied.

Fractal Box Counting Technique

Root system structures of heterogeneous plant communities were recorded as integral systems by using the trench profile method. Fractal dimensions of the root images were calculated from image files by the box-counting method (2, 4, 8, 16, 32, 64, 80, 128, 200, 256). This method allows the structural complexity of such associations to be compared between plant communities, with regard to their potentials for soil resource acquisition and utilization. The digitized root images were used for fractal analysis following the box-counting method described by (Tatsumi et al., 1989) and (Ketipearachchi and Tatsumi, 2000; Klarizze, 2005; Margitta and Arthur, 2005).

The rooting patterns of the root system associations of different plant communities were analysed and quantified as integral basic functional units. The box-counting method (Block et al., 1990; Diebel and Feret, 1993; Jurgens et al., 1996; Ketipearachchi and Tatsumi, 2000) was applied to determine the fractal dimension. The scanned images of root system associations were first covered with a frame. The frame was divided into a grid, each box having a side length 'a'. The box size changed within a range of 5–20 mm (corresponding to a range of 50–200 mm for the original profile) in three steps. The number of intersected boxes *N (a)*, found in the root images at each scale were counted. Plotting the number of boxes *N (a)* against side length *a* on a log–log scale gave a straight line. The slope of the line was used to calculate the fractal dimension (D) which results from the quotient of two logarithms following the power-law relationship:

$N = a^D \rightarrow D = \log N / \log a$.

In this paper we used: $N \propto L^{-D}$, $\log N = \log c - D \log L$

L= Box size, N= # of Box count the intersected.

Results and Discussion

General clonal growth patterns

Scirpus grossus plant reiterates by rhizomatous growth and branches from a single mother plant. Invariably, fertilizer application has led to faster reiteration than those devoid of fertilizer application in both peat and paddy soils with more robust growth of ramets (Table 1). The resultant populations were more robust aerial plant growth with *ca.* 253.5 ramets m^{-2} (mean dry aerial bio mass of 23.2 g $plant^{-1}$) compared with 235.6 ramets m^{-2} (16.3 g $plant^{-1}$) in unfertilized peat soils 24 weeks after planting of the mother plant. The parallel figures for plants growing on paddy soils of the Jawa series were *ca.* 97.08 ramets m^{-1} (12.19 g $plant^{-1}$) (fertilized paddy soils) and 83.67 ramets m^{-1} (10.89 g $plant^{-1}$)(unfertilized paddy soils) 24 weeks after planting of the mother plant (Table 1, Figure 2). Mean ramets mortality was significantly higher in unfertilized paddy soils at 121.3 ramets m^{-2}, while in the fertilized paddy soils this was only 34.7 ramets m^{-2}, resulting respective net populations of *ca.* 218.8 ramets m^{-2} and 114.3 ramets m^{-2} in fertilized and unfertilized plots. In paddy soils mean ramets mortality in unfertilized paddy soils was *ca.* 8.58 ramets m^{-2}, while this was only *ca.* 5.67 ramets m$^{-2,}$ leading to the respective resultant net populations of 91.41m^{-2} and 75.09 ramets m^{-2} (Table 1 and Figure 2). Flowering set in earlier among ramets in fertilized peat soils with 103.2 ramets m^{-2}*vis-a-vis* 77.5 ramets m^{-2}, 24 weeks after transplanting of the mother plant in unfertilized soil (Figure 2).

Dualism in growth strategy and resource capture

Scirpus grossus displayed dualism in resource capture by having both guerrilla (through the lateral proliferation of rhizomatous ramets) (Figures 3a, 3b and 4a, 4b). This distinct capacity gives *S. grossus* the edge advantage in the presence of competing neighbours for lateral resource capture. The initial analyses indicated that fertilizer applications of NPK at 100:30:30 has led to more robust lateral growths, albeit small in both peat and paddy soils compared with the respective controls. Further assessment of optimum resource capture based on image analysis on time-mediated lateral growths of modules and fractal dimensions are on going.

Fractal dimension analyses

Root growth is effected to the acquisition, distribution, and consumption of water and nutrients of plants. If we want to understand root branching complexity, mathematical models using fractals and computers can be applied to accurately understand the growth and form complexity of plant root systems. Because these we can using the Fractal Dimensions software's box-counting method of fractal analyses. The data from the fractal counting were tabulated and plotted on a log-log plot graph. Fractal dimension analysis of subterranean rhizomes by Image J program on *S. grossus* in peat and paddy soils do not found in partly later not significant differences in peat soils (fractal dimension between 1.52 ± 0.53 and 1.50 ± 0.59) in unfertilized and fertilized plots. In addition, in paddy soil, fractal dimension readings were between 1.53 ± 0.55 and 1.52 ± 0.49) in unfertilized and fertilized plots (Tables 2 and 3, Figures 5a,

Fig. 2. Scirpus grossus plants 6 months after sowing on (A) paddy soils and (B) peat soils

Table 1
General growth patterns of *Scirpus grossus* in unfertilized soil (F_0) and fertilized (F_1) peat and paddy soils

Growth parameters**	Unfertilized peat soil	Fertilized peat soil
Gross plant number	117.83 m^{-2} b (83.67)d	126.75 m^{-2} a (97.08)c
Mortality number	30.33 m^{-2} a (8.58)b	8.67 m^{-2} b (5.67)b
Net plant number	87.5 m^{-2} b (75.09)c*	118.08 m^{-2} a (91.41)b
Flowers number	38.75 m^{-2} b (16.42)d*	51.58 m^{-2} a (23.67) cd
Leaf weight, g	6.9c (12.7)ab	9.7bc (14.8)a
Stem weight, g	7.9b (17.6)a	10.5b (8.00)b
Florescence, g	1.9c (2.38) abc*	2.8ab (3.13)a

* Paddy soils in parentheses. ** Values in a row followed by a common uppercase letter are not significantly different at p<0.01(HSD tests). * Paddy soils in parentheses.

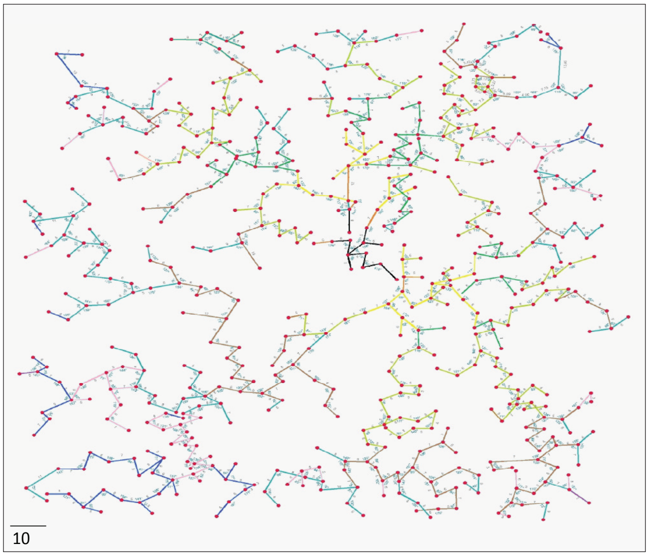

Fig. 3a. Time-mediated growth of subterranean rhizomes of *Scirpus grossus* in fertilizer peat soil (F_0) 6 months after planting of mother plant.

Blake, 1st, 2nd week; Orange, 3rd, 4th week; Yellow, 5th and 6th week; Dark green, 7th and 8th week.; Olive green 9th, 10th week; Blue, 11th, 12th week; Brawn, 13th, 14th week; Pink, 15th, 16th week; Dark blue, 17th, 18th week; Purple, 19th, 20th week; Grey, 21th, 22th week

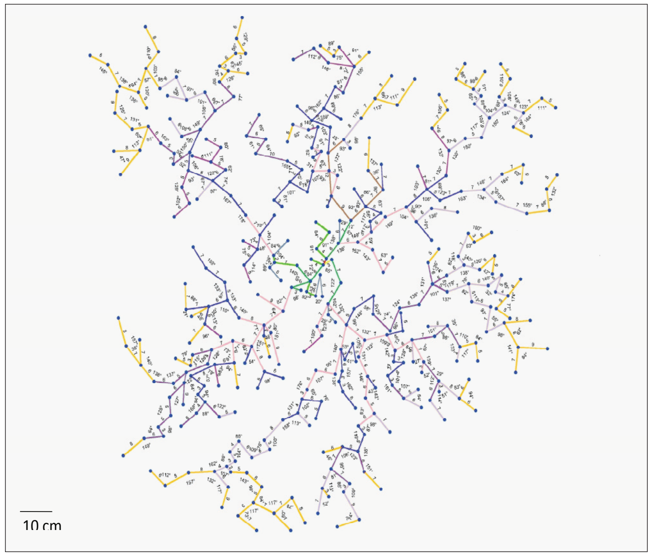

Fig. 4a. Time-mediated growth of subterranean rhizomes of *Scirpus grossus* in fertilizer paddy soil (F_0) 6 months after planting of mother plant.

Blake, 1st, 2nd week; Orange, 3rd, 4th week; Yellow, 5th and 6th week; Dark green, 7th and 8th week.; Olive green 9th, 10th week; Blue, 11th, 12th week; Brawn, 13th, 14th week; Pink, 15th, 16th week; Dark blue, 17th, 18th week; Purple, 19th, 20th week; Grey, 21th, 22th week

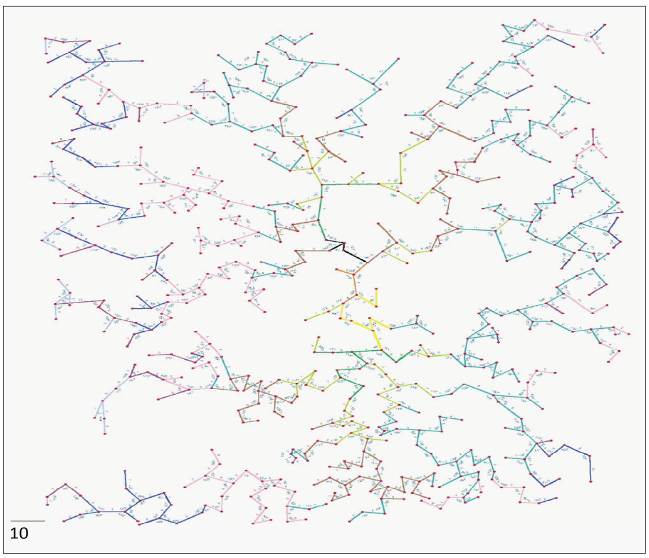

Fig. 3b. Time-mediated growth of subterranean rhizomes of *Scirpus grossus* in fertilizer peat soil (F_1) 6 months after planting of mother plant.

Blake, 1st, 2nd week; Orange, 3rd, 4th week; Yellow, 5th and 6th week; Dark green, 7th and 8th week.; Olive green 9th, 10th week; Blue, 11th, 12th week; Brawn, 13th, 14th week; Pink, 15th, 16th week; Dark blue, 17th, 18th week; Purple, 19th, 20th week; Grey, 21th,22th week

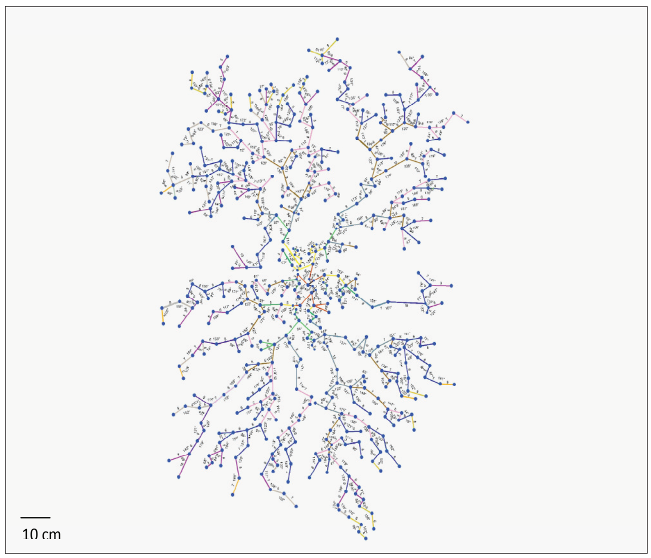

Fig. 4b. Time-mediated growth of subterranean rhizomes of *Scirpus grossus* in fertilizer paddy soil (F_1) 6 months after planting of mother plant.

Blake, 1st, 2nd week; Orange, 3rd, 4th week; Yellow, 5th and 6th week; Dark green, 7th and 8th week.; Olive green 9th, 10th week; Blue, 11th, 12th week; Brawn, 13th, 14th week; Pink, 15th, 16th week; Dark blue, 17th, 18th week; Purple, 19th, 20th week; Grey, 21th, 22th week b and 6). However, we found in early partly significant differences be compared between plant in peat at 12 weeks in fertilized plots we can saw tow direction growth plant, but do not saw in unfertilized. In paddy soils, we saw tow direction in both plots unfertilized and fertilized. However, we found in fertilized more density (Tables 2 and 3, Figures 5a, b and 6). In addition, when we compared between plants in peat and paddy soils we found in peat soil more distances inter plants but in paddy soils

Table 2
Fractal dimensions of *Scirpus grossus* in unfertilized and fertilized peat soils.

BOX SIZE/COUNT	4W	8W	12W	16W	20W	24W
2	90	674	4292	8173	8909	8909
4	36	284	1811	3437	3700	3700
8	17	118	796	1512	1608	1608
16	7	54	331	632	691	691
32	3	24	122	236	251	251
64	1	9	41	71	72	72
80	1	6	29	49	52	52
128	1	4	13	20	20	20
200	1	1	7	9	9	9
256	1	1	4	6	6	6
D_{CA}	0.99	1.34	1.43	1.50	1.52	1.52
D_{FA}	0.94	1.18	1.35	1.46	1.49	1.50

*D_{CA}: Dimension control area A. * D_{FA}: Dimension fertilizer area A.

Table 3
Fractal dimensions of *Scirpus grossus* in unfertilized and fertilized paddy soils

BOX SIZE/COUNT	4W	8W	12W	16W	20W	24W
2	86	189	4292	8173	8909	8909
4	33	284	1811	3437	3700	3700
8	17	118	796	1512	1608	1608
16	6	54	331	632	691	691
32	2	24	122	236	251	251
64	1	9	41	71	72	72
80	1	6	29	49	52	52
128	1	4	13	20	20	20
200	1	1	7	9	9	9
256	1	1	4	6	6	6
D_{CB}	0.98	1.34	1.43	1.50	1.52	1.52
D_{FB}	1.03	1.18	1.35	1.46	1.49	1.50

*D_{CB}: Dimension control area B. *D_{FB}: Dimension fertilizer area B.

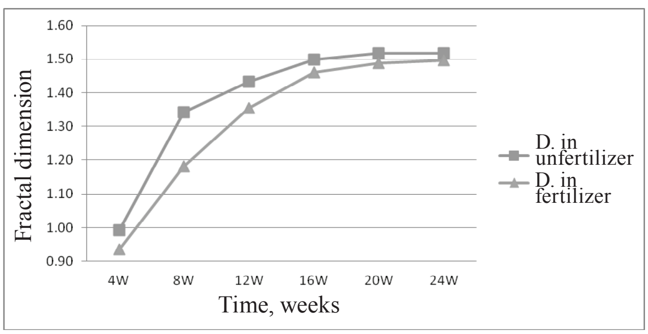

Fig 5a. Fractal dimensions of *Scirpus grossus* in unfertilized and fertilized in peat soils

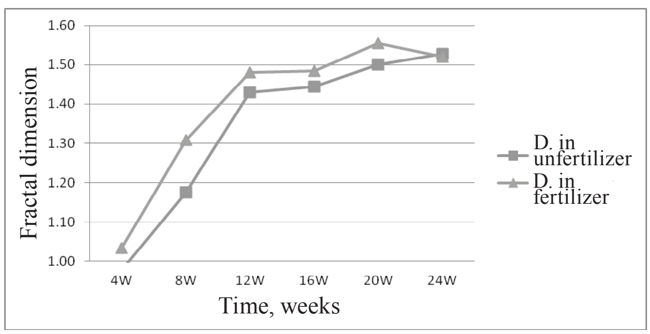

Fig. 5b. Fractal dimensions of *Scirpus grossus* in unfertilized and fertilized in paddy soils

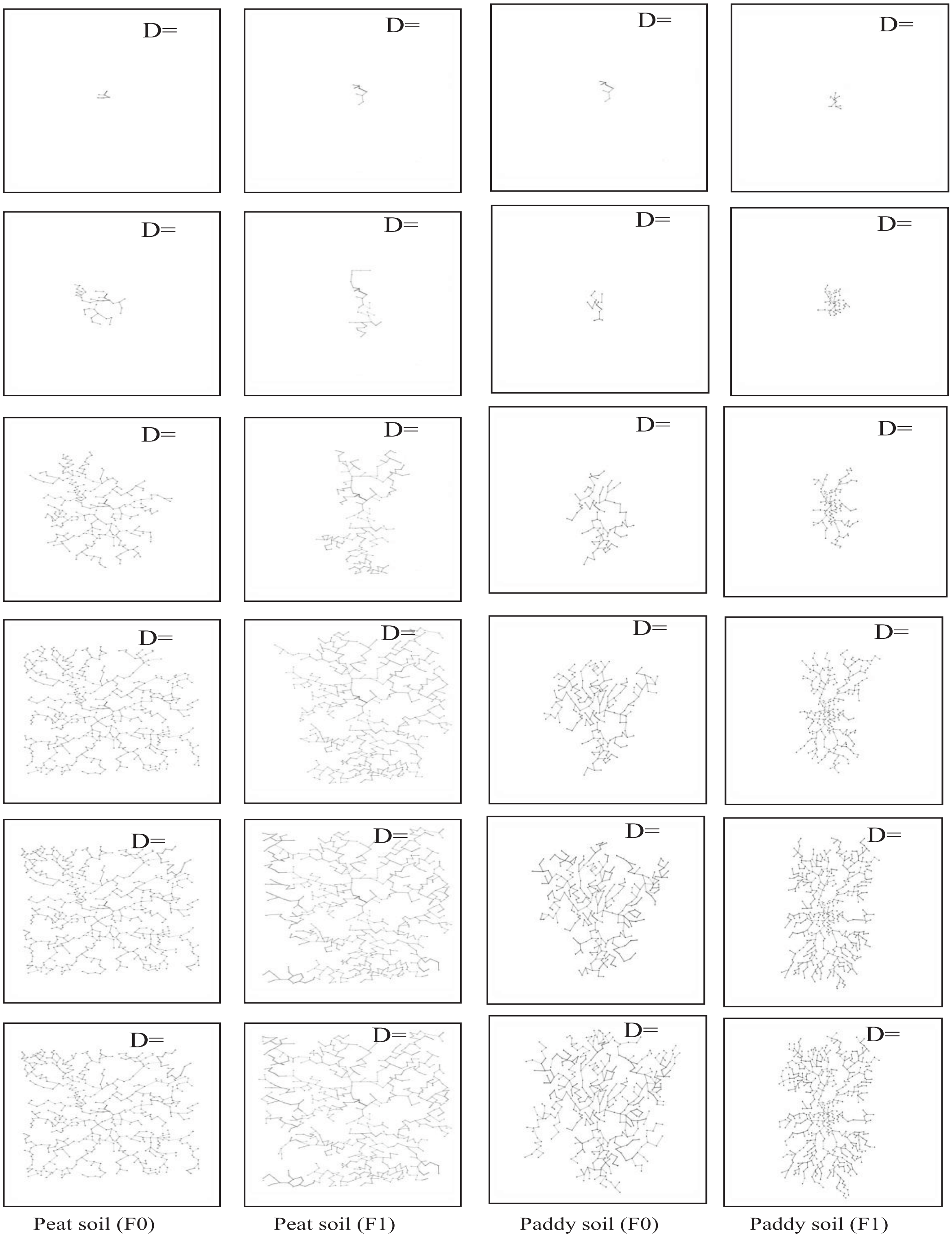

Fig. 6. Fractal dimensions with time-mediated growth of subterranean rhizomes of *Scirpus grossus* in unfertilized and fertilized peat and paddy soils. F0 - unfertilized soils; F1 - fertilized soils.

we found more density plants (Figure 6). Similar method was used to examine the developmental responses of root systems in upland rice genotype (Margitta amd Arthur, 2005) with fractal dimensions of the root images were calculated. In addition, in study, root system structures of heterogeneous plant communities were recorded as integral systems by using the trench profile method. Fractal dimensions of the root images were calculated from image files by the box-counting method. They found similar results (Margitta and Arthur, 2005).

Conclusions

In this study, the results showed the presence of minor differences in rhizome growth patterns between fertilized and unfertilized plants. In addition, the results showed the presence of minor differences in rhizome growth patterns between fertilized and unfertilized plants in peat and paddy soils. Fractals are useful in analyzing complex biological systems accurately. The fractal dimension (D) served as the summary statistic of the branching characteristics of *Scirpus grossus* L. The fractal dimension could be of interest to botanists and management and control of weed because it is directly correlated with the efficiency at which the roots exploit soil resources. The use of soil nutrients and the application of other forms of environmental stress, perhaps are also useful in such an endeavour to further understand behavioural patterns of root growth in heterogeneous environments.

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